Analysis of Covariance

Analysis of covariance

- ANOVA: explanatory variables categorical (divide data into groups)
- traditionally, analysis of covariance has categorical x's plus one numerical x ("covariate") to be adjusted for.
- `lm` handles this too.
- Simple example: two treatments (drugs) (`a` and `b`), with before and after scores.
- Does knowing before score and/or treatment help to predict after score?
- Is after score different by treatment/before score?

Data

Treatment, before, after:

```
a 5 20
a 10 23
a 12 30
a 9 25
a 23 34
a 21 40
a 14 27
a 18 38
a 6 24
a 13 31
b 7 19
b 12 26
b 27 33
b 24 35
b 18 30
b 22 31
b 26 34
```

Packages

```
library(tidyverse)
library(broom)
library(marginaleffects)
```

the last of these for predictions.

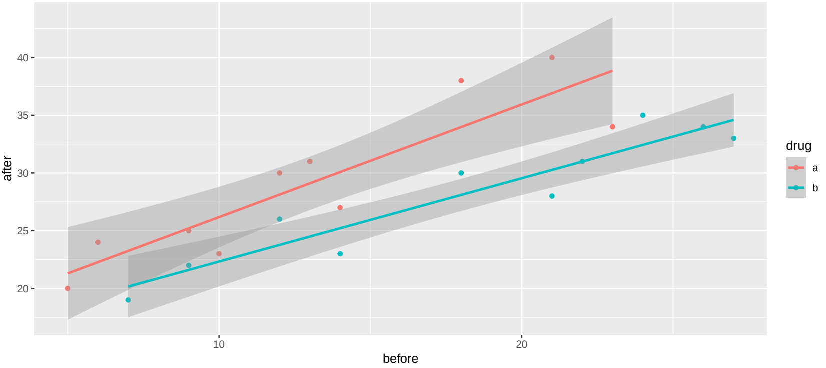
Read in data

```
url <- "http://ritsokiguess.site/datafiles/ancova.txt"
prepost <- read_delim(url, " ")
prepost
```

```
# A tibble: 20 x 3
   drug  before after
   <chr>  <dbl> <dbl>
 1 a          5    20
 2 a         10    23
 3 a         12    30
 4 a          9    25
 5 a         23    34
 6 a         21    40
 7 a         14    27
 8 a         18    38
 9 a          6    24
10 a         13    31
11 b          7    19
```

Making a plot

```
ggplot(prepost, aes(x = before, y = after, colour = drug)) +
  geom_point() + geom_smooth(method = "lm")
```



Comments

- As before score goes up, after score goes up.
- Red points (drug A) generally above blue points (drug B), for comparable before score.
- Suggests before score effect *and* drug effect.

The means

```
prepost %>%
  group_by(drug) %>%
  summarize(
    before_mean = mean(before),
    after_mean = mean(after)
  )
```

```
# A tibble: 2 x 3
  drug  before_mean after_mean
  <chr>       <dbl>      <dbl>
1 a            13.1       29.2
2 b            18         28.1
```

- Mean “after” score slightly higher for treatment A.
- Mean “before” score much higher for treatment B.
- Greater *improvement* on treatment A.

Testing for interaction

```r
prepost.1 <- lm(after ~ before * drug, data = prepost)
anova(prepost.1)
```

```
Analysis of Variance Table

Response: after
            Df Sum Sq Mean Sq F value    Pr(>F)
before       1 430.92  430.92 62.6894  6.34e-07 ***
drug         1 115.31  115.31 16.7743 0.0008442 ***
before:drug  1  12.34   12.34  1.7948 0.1990662
Residuals   16 109.98    6.87
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

```r
summary(prepost.1)
```

```

Call:
lm(formula = after ~ before * drug, data = prepost)
```

Predictions

Set up values to predict for:

```
summary(prepost)
```

```
     drug               before          after      
 Length:20          Min.   : 5.00   Min.   :19.00  
 Class :character   1st Qu.: 9.75   1st Qu.:23.75  
 Mode  :character   Median :14.00   Median :29.00  
                    Mean   :15.55   Mean   :28.65  
                    3rd Qu.:21.25   3rd Qu.:33.25  
                    Max.   :27.00   Max.   :40.00  
```

```
new <- datagrid(before = c(9.75, 14, 21.25),
                drug = c("a", "b"), model = prepost.1)
new
```

```
  before drug rowid
1   9.75    a     1
2   9.75    b     2
3  14.00    a     3
```

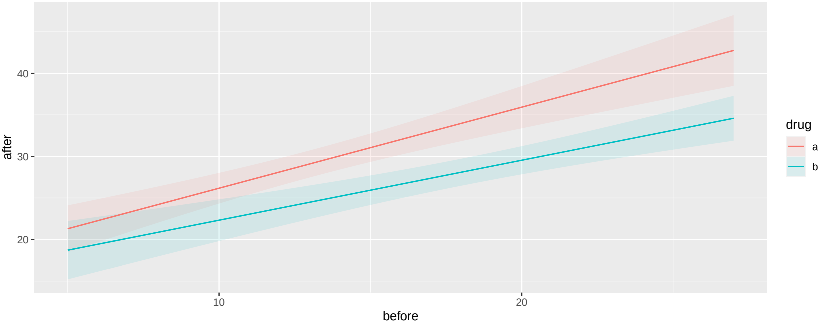
and then

```
cbind(predictions(prepost.1, newdata = new)) %>%
  select(drug, before, estimate, conf.low, conf.high)
```

```
  drug before estimate conf.low conf.high
1    a   9.75 25.93250 24.05059  27.81442
2    b   9.75 22.14565 19.58681  24.70450
3    a  14.00 30.07784 28.43296  31.72271
4    b  14.00 25.21304 23.32649  27.09959
5    a  21.25 37.14929 34.32557  39.97300
6    b  21.25 30.44565 28.64373  32.24758
```

Predictions (with interaction included), plotted

```
plot_predictions(model = prepost.1, condition = c("before",
```



Lines almost parallel, but not quite.

Taking out interaction

```
prepost.2 <- update(prepost.1, . ~ . - before:drug)
summary(prepost.2)
```

```

Call:
lm(formula = after ~ before + drug, data = prepost)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6348 -2.5099 -0.2038  1.8871  4.7453 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  18.3600     1.5115  12.147 8.35e-10 ***
before        0.8275     0.0955   8.665 1.21e-07 ***
drugb        -5.1547     1.2876  -4.003 0.000921 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.682 on 17 degrees of freedom
Multiple R-squared:  0.817, Adjusted R-squared:  0.7955
```

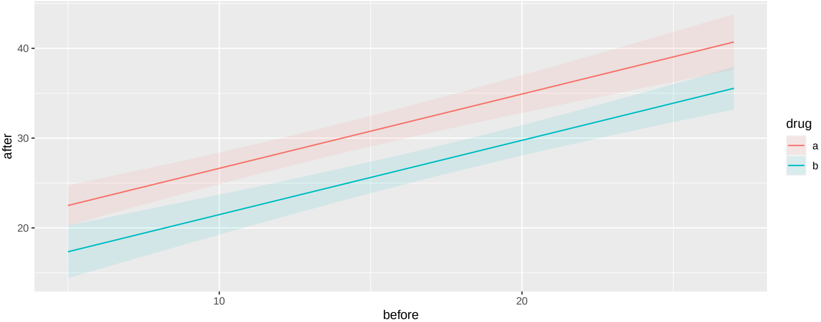
Predictions

```
cbind(predictions(prepost.2, newdata = new)) %>%
  select(drug, before, estimate)
```

```
  drug before estimate
1    a   9.75 26.42794
2    b   9.75 21.27328
3    a  14.00 29.94473
4    b  14.00 24.79007
5    a  21.25 35.94397
6    b  21.25 30.78931
```

Plot of predicted values

```
plot_predictions(prepost.2, condition = c("before", "drug")
```



This time the lines are *exactly* parallel. No-interaction model forces them to have the same slope.

Different look at model output

- `anova(prepost.2)` tests for significant effect of before score and of drug, but doesn't help with interpretation.
- `summary(prepost.2)` views as regression with slopes:

```
summary(prepost.2)
```

```
Call:
lm(formula = after ~ before + drug, data = prepost)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6348 -2.5099 -0.2038  1.8871  4.7453 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  18.3600     1.5115  12.147 8.35e-10 ***
before        0.8275     0.0955   8.665 1.21e-07 ***
drugb        -5.1547     1.2876  -4.003 0.000921 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.682 on 17 degrees of freedom
Multiple R-squared:  0.817,	Adjusted R-squared:  0.7955 
F-statistic: 37.96 on 2 and 17 DF,  p-value: 5.372e-07
```

Understanding those slopes

```
tidy(prepost.2)
```

```
# A tibble: 3 x 5
  term        estimate std.error statistic  p.value
  <chr>          <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)   18.4      1.51       12.1  8.35e-10
2 before         0.827    0.0955      8.66 1.21e- 7
3 drugb         -5.15     1.29       -4.00 9.21e- 4
```

- `before` ordinary numerical variable; `drug` categorical.
- `lm` uses first category `druga` as baseline.
- Intercept is prediction of after score for before score 0 and *drug A*.
- `before` slope is predicted change in after score when before score increases by 1 (usual slope)
- Slope for `drugb` is *change* in predicted after score for being on drug B rather than drug A. Same for *any* before score (no interaction).

Summary

- ANCOVA model: fits different regression line for each group, predicting response from covariate.
- ANCOVA model with interaction between factor and covariate allows different slopes for each line.
- Sometimes those lines can cross over!
- If interaction not significant, take out. Lines then parallel.
- With parallel lines, groups have consistent effect regardless of value of covariate.