# Case study: asphalt

# The asphalt data

- 31 asphalt pavements prepared under different conditions. How does quality of pavement depend on these?
- Variables:
  - `pct.a.surf` Percentage of asphalt in surface layer
  - `pct.a.base` Percentage of asphalt in base layer
  - `fines` Percentage of fines in surface layer
  - `voids` Percentage of voids in surface layer
  - `rut.depth` Change in rut depth per million vehicle passes
  - `viscosity` Viscosity of asphalt
  - `run` 2 data collection periods: 1 for run 1, 0 for run 2.
- `rut.depth` response. Depends on other variables, how?

# Packages for this section

```
library(MASS)
library(tidyverse)
library(broom)
library(leaps)
```

Make sure to load `MASS` before `tidyverse` (for annoying technical reasons).

# Getting set up

```
my_url <- "http://ritsokiguess.site/datafiles/asphalt.txt"
asphalt <- read_delim(my_url, " ")
```

- Quantitative variables with one response: multiple regression.
- Some issues here that don't come up in "simple" regression; handle as we go. (STAB27/STAC67 ideas.)

## The data (some)

```
asphalt
```

```
# A tibble: 31 x 7
   pct.a.surf pct.a.base fines voids rut.depth viscosity
        <dbl>      <dbl> <dbl> <dbl>     <dbl>     <dbl> <c
 1       4.68       4.87   8.4  4.92      6.75      2.8
 2       5.19       4.5    6.5  4.56     13         1.4
 3       4.82       4.73   7.9  5.32     14.8       1.4
 4       4.85       4.76   8.3  4.86     12.6       3.3
 5       4.86       4.95   8.4  3.78      8.25      1.7
 6       5.16       4.45   7.4  4.40     10.7       2.9
 7       4.82       5.05   6.8  4.87      7.28      3.7
 8       4.86       4.7    8.6  4.83     12.7       1.7
 9       4.78       4.84   6.7  4.86     12.6       0.92
10       5.16       4.76   7.7  4.03     20.6       0.68
# i 21 more rows
```

# Plotting response "rut depth" against everything else

Same idea as for plotting separate predictions on one plot:

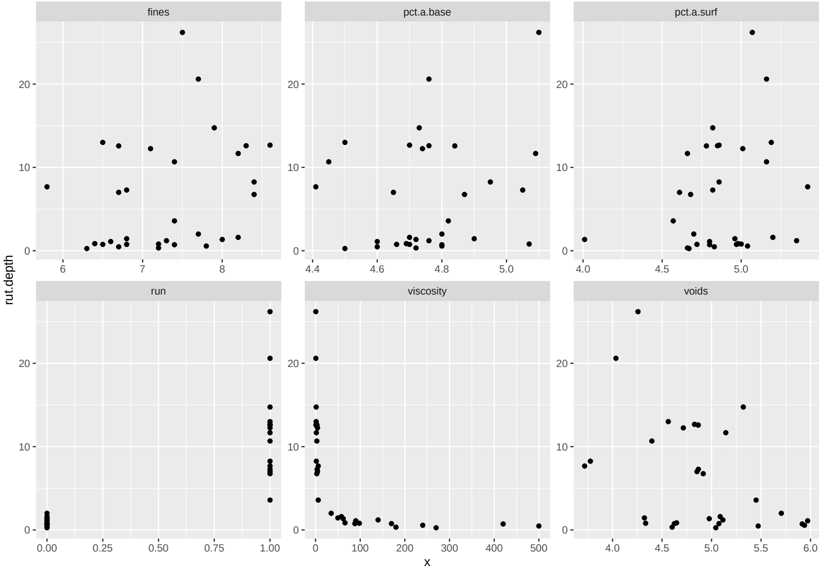
```
asphalt %>%
  pivot_longer(
    -rut.depth,
    names_to="xname", values_to="x"
  ) %>%
  ggplot(aes(x = x, y = rut.depth)) + geom_point() +
  facet_wrap(~xname, scales = "free") -> g
```

"collect all the x-variables together into one column called x, with another column xname saying which x they were, then plot these x's against rut.depth, a separate facet for each x-variable."

I saved this graph to plot later (on the next page).

# The plot

```
g
```



fines
pct.a.base
pct.a.surf
run
viscosity
voids
rut.depth
x
0
10
20
6
7
8
4.4
4.6
4.8
5.0
4.0
4.5
5.0
0.00
0.25
0.50
0.75
1.00
0
100
200
300
400
500
4.0
4.5
5.0
5.5
6.0

# Interpreting the plots

- One plot of rut depth against each of the six other variables.
- Get rough idea of what's going on.
- Trends mostly weak.
- `viscosity` has strong but non-linear trend.
- `run` has effect but variability bigger when run is 1.
- Weak but downward trend for `voids`.
- Non-linearity of `rut.depth-viscosity` relationship should concern us.

# Log of `viscosity`: more nearly linear?

- Take this back to asphalt engineer: suggests log of `viscosity`:

```
ggplot(asphalt, aes(y = rut.depth, x = log(viscosity))) +
  geom_point() + geom_smooth(se = F) -> g
```

(plot overleaf)

# Rut depth against log-viscosity

## Comments and next steps

- Not very linear, but better than before.
- In multiple regression, hard to guess which x's affect response. So typically start by predicting from everything else.
- Model formula has response on left, squiggle, explanatories on right joined by plusses:

```
rut.1 <- lm(rut.depth ~ pct.a.surf + pct.a.base + fines +
  voids + log(viscosity) + run, data = asphalt)
summary(rut.1)
```

```
Call:
lm(formula = rut.depth ~ pct.a.surf + pct.a.base + fines +
    log(viscosity) + run, data = asphalt)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.1211 -1.9075 -0.7175  1.6382  9.5947 
```

# Regression output: `summary(rut.1)` or:

```
glance(rut.1)
```

```
# A tibble: 1 x 12
  r.squared adj.r.squared sigma statistic     p.value    df logLik   AI
      <dbl>         <dbl> <dbl>     <dbl>       <dbl> <dbl>  <dbl> <dbl
1     0.806         0.758  3.32      16.6 0.000000174     6  -77.3  171
# i 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
tidy(rut.1)
```

```
# A tibble: 7 x 5
  term           estimate std.error statistic p.value
  <chr>             <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)     -13.0      26.2      -0.496 0.625
2 pct.a.surf        3.97      2.50      1.59  0.125
3 pct.a.base        1.26      3.97      0.318 0.753
4 fines             0.116     1.01      0.115 0.909
5 voids             0.589     1.32      0.445 0.660
6 log(viscosity)   -3.15      0.919    -3.43  0.00220
7 run              -1.97      3.65     -0.539 0.595
```

# Comments

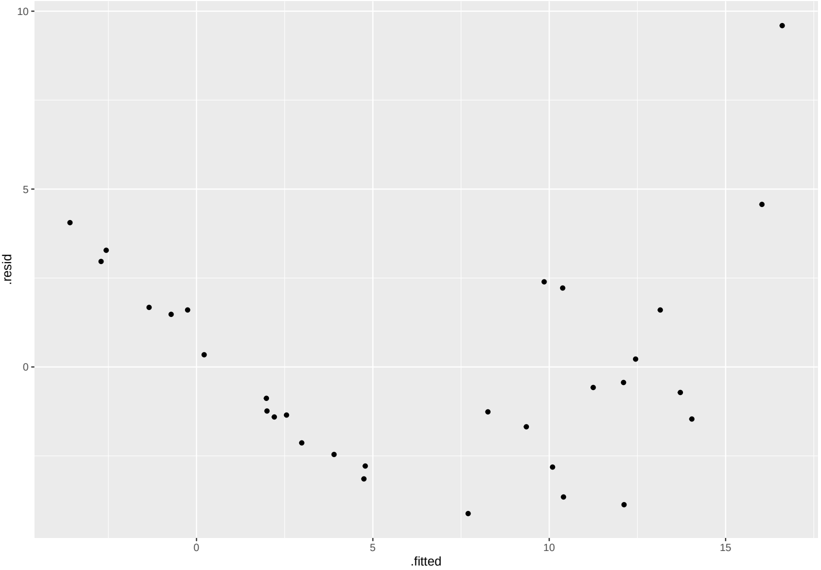
- R-squared 81%, not so bad.
- P-value in `glance` asserts that something helping to predict rut.depth.
- Table of coefficients says `log(viscosity)`.
- But confused by clearly non-significant variables: remove those to get clearer picture of what is helpful.
- **Before we do anything, look at residual plots:**

  `(a) of residuals against fitted values (as usual)`

  - (b) of residuals against each explanatory.
- Problem fixes:
  - with (a): fix response variable;
  - with some plots in (b): fix those explanatory variables.

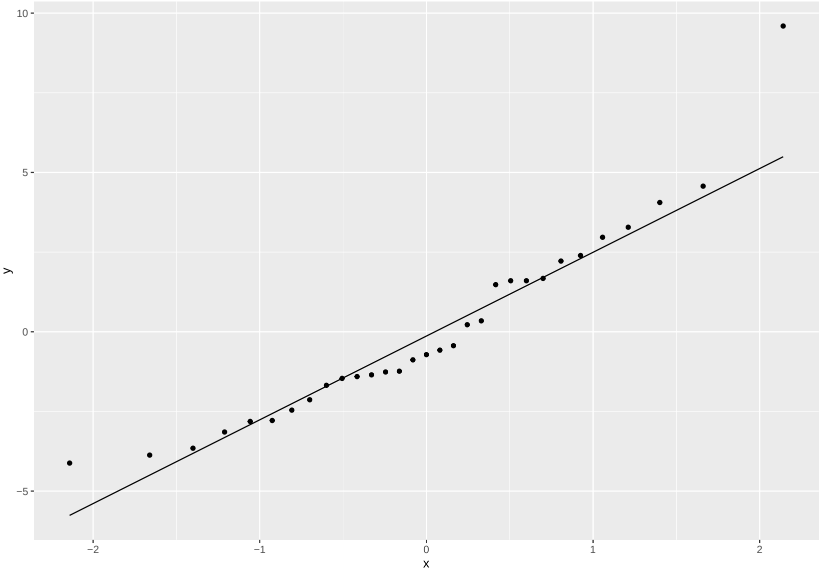
# Plot fitted values against residuals

```
ggplot(rut.1, aes(x = .fitted, y = .resid)) + geom_point()
```



# Normal quantile plot of residuals

```
ggplot(rut.1, aes(sample = .resid)) + stat_qq() + stat_qq_l
```



# Plotting residuals against $x$ variables

- Problem here is that residuals are in the fitted model, and the observed $x$-values are in the original data frame `asphalt`.
- Package broom contains a function `augment` that combines these two together so that they can later be plotted: start with a model first, and then augment with a data frame:

```
rut.1 %>% augment(asphalt) -> rut.1a
```

# What does rut.1a contain?

```
names(rut.1a)
```

```
 [1] "pct.a.surf" "pct.a.base" "fines"      "voids"      "r
 [6] "viscosity"  "run"        ".fitted"    ".resid"     ".
[11] ".sigma"     ".cooksd"    ".std.resid"
```

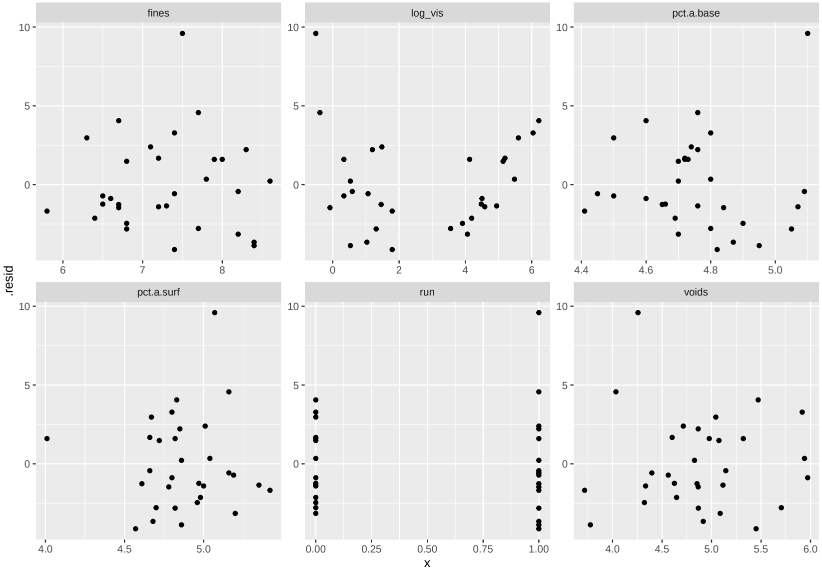
- all the stuff in original data frame, plus:
- quantities from regression (starting with a dot)

# Plotting residuals against $x$-variables

```
rut.1a %>%
  mutate(log_vis=log(viscosity)) %>%
  pivot_longer(
    c(pct.a.surf:voids, run, log_vis),
    names_to="xname", values_to="x"
  ) %>%
  ggplot(aes(x = x, y = .resid)) +
  geom_point() + facet_wrap(~xname, scales = "free") -> g
```

# The plot

```
g
```



# Comments

- There is serious curve in plot of residuals vs. fitted values. Suggests a transformation of $y$.
- The residuals-vs-$x$'s plots don't show any serious trends. Worst probably that potential curve against log-viscosity.
- Also, large positive residual, 10, that shows up on all plots. Perhaps transformation of $y$ will help with this too.
- If residual-fitted plot OK, but some residual-$x$ plots not, try transforming those $x$'s, eg. by adding $x^2$ to help with curve.

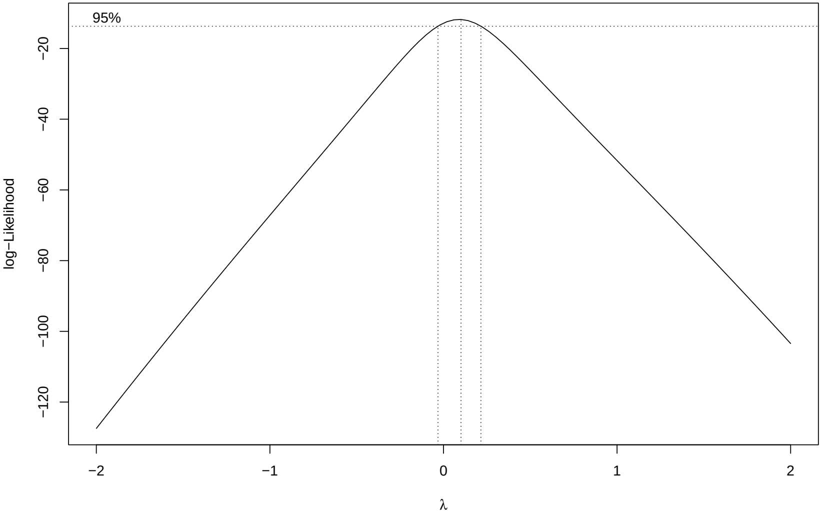
# Which transformation?

- Best way: consult with person who brought you the data.
- Can't do that here!
- No idea what transformation would be good.
- Let data choose: "Box-Cox transformation".
- Scale is that of "ladder of powers": power transformation, but 0 is log.

# Running Box-Cox

From package `MASS`:

```
boxcox(rut.depth ~ pct.a.surf + pct.a.base + fines + voids
  log(viscosity) + run, data = asphalt)
```



# Comments on Box-Cox plot

- $\lambda$ represents power to transform $y$ with.
- Best single choice of transformation parameter $\lambda$ is peak of curve, close to 0.
- Vertical dotted lines give CI for $\lambda$, about (−0.05, 0.2).
- $\lambda = 0$ means "log".
- Narrowness of confidence interval mean that these not supported by data:
  - No transformation ($\lambda = 1$)
  - Square root ($\lambda = 0.5$)
  - Reciprocal ($\lambda = -1$).

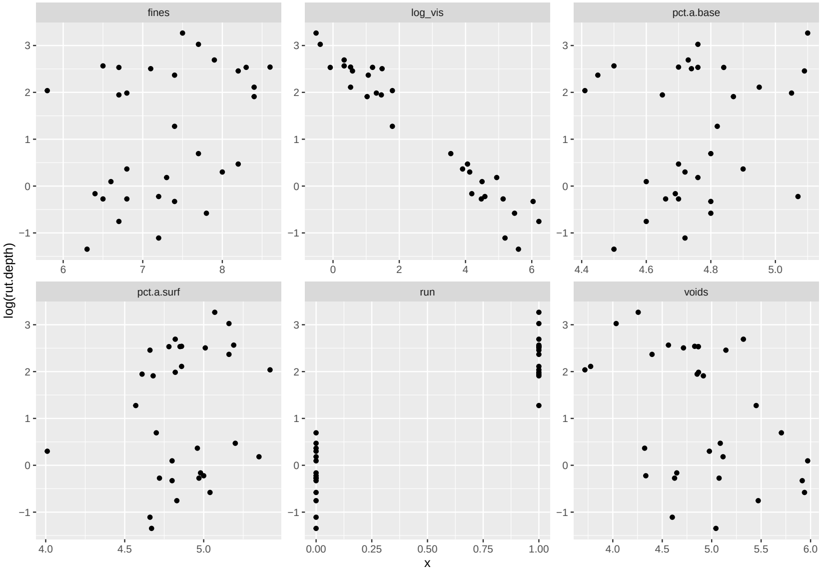
# Relationships with explanatories

- As before: plot response (now `log(rut.depth)`) against other explanatory variables, all in one shot:

```
asphalt %>%
  mutate(log_vis=log(viscosity)) %>%
  pivot_longer(
    c(pct.a.surf:voids, run, log_vis),
    names_to="xname", values_to="x"
  ) %>%
  ggplot(aes(y = log(rut.depth), x = x)) + geom_point() +
  facet_wrap(~xname, scales = "free") -> g3
```

# The new plots

```
g3
```



# Modelling with transformed response

- These trends look pretty straight, especially with `log.viscosity`.
- Values of `log.rut.depth` for each `run` have same spread.
- Other trends weak, but are straight if they exist.
- Start modelling from the beginning again.
- Model `log.rut.depth` in terms of everything else, see what can be removed:

```
rut.2 <- lm(log(rut.depth) ~ pct.a.surf + pct.a.base +
  fines + voids + log(viscosity) + run, data = asphalt)
```

- use `tidy` from `broom` to display just the coefficients.

# Output

```r
tidy(rut.2)
```

```
# A tibble: 7 x 5
  term             estimate std.error statistic     p.value
  <chr>               <dbl>     <dbl>     <dbl>       <dbl>
1 (Intercept)       -1.57      2.44      -0.646 0.525
2 pct.a.surf         0.584     0.232      2.52  0.0190
3 pct.a.base        -0.103     0.369     -0.280 0.782
4 fines              0.0978    0.0941     1.04  0.309
5 voids              0.199     0.123      1.62  0.119
6 log(viscosity)    -0.558     0.0854    -6.53  0.000000945
7 run                0.340     0.339      1.00  0.326
```

```r
summary(rut.2)
```

```

Call:
lm(formula = log(rut.depth) ~ pct.a.surf + pct.a.base + fi
    voids + log(viscosity) + run, data = asphalt)
```

# Taking out everything non-significant

- Try: remove everything but pct.a.surf and log.viscosity:

```
rut.3 <- lm(log(rut.depth) ~ pct.a.surf + log(viscosity), data = asphal
summary(rut.3)
```

```
Call:
lm(formula = log(rut.depth) ~ pct.a.surf + log(viscosity), data = aspha

Residuals:
     Min       1Q   Median       3Q      Max 
-0.61938 -0.21361  0.06635  0.14932  0.63012 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)     0.90014    1.08059   0.833   0.4119    
pct.a.surf      0.39115    0.21879   1.788   0.0846 .  
log(viscosity) -0.61856    0.02713 -22.797   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3208 on 28 degrees of freedom
Multiple R-squared:  0.9509,    Adjusted R-squared:  0.9474
```

## Find the largest P-value by eye:

```
tidy(rut.2)
```

```
# A tibble: 7 x 5
  term             estimate std.error statistic     p.value
  <chr>               <dbl>     <dbl>     <dbl>       <dbl>
1 (Intercept)       -1.57      2.44      -0.646 0.525
2 pct.a.surf         0.584     0.232      2.52  0.0190
3 pct.a.base        -0.103     0.369     -0.280 0.782
4 fines              0.0978    0.0941     1.04  0.309
5 voids              0.199     0.123      1.62  0.119
6 log(viscosity)    -0.558     0.0854    -6.53  0.000000945
7 run                0.340     0.339      1.00  0.326
```

- Largest P-value is 0.78 for `pct.a.base`, not significant.
- So remove this first, re-fit and re-assess.
- Or, as over.

# Get the computer to find the largest P-value for you

- Output from `tidy` is itself a data frame, thus:

```
tidy(rut.2) %>% arrange(p.value)
```

```
# A tibble: 7 x 5
  term             estimate std.error statistic     p.value
  <chr>               <dbl>     <dbl>     <dbl>       <dbl>
1 log(viscosity)    -0.558     0.0854    -6.53  0.000000945
2 pct.a.surf         0.584     0.232      2.52  0.0190
3 voids              0.199     0.123      1.62  0.119
4 fines              0.0978    0.0941     1.04  0.309
5 run                0.340     0.339      1.00  0.326
6 (Intercept)       -1.57      2.44      -0.646 0.525
7 pct.a.base        -0.103     0.369     -0.280 0.782
```

- Largest P-value at the bottom.

# Take out pct.a.base

- Copy and paste the `lm` code and remove what you're removing:

```
rut.4 <- lm(log(rut.depth) ~ pct.a.surf + fines + voids +
                 log(viscosity) + run, data = asphalt)
tidy(rut.4) %>% arrange(p.value) %>% dplyr::select(term, p.value
```

```
# A tibble: 6 x 2
  term                  p.value
  <chr>                   <dbl>
1 log(viscosity) 0.000000448
2 pct.a.surf     0.0143
3 voids          0.109
4 (Intercept)    0.208
5 run            0.279
6 fines          0.316
```

- `fines` is next to go, P-value 0.32.

# “Update”

Another way to do the same thing:

```
rut.4 <- update(rut.2, . ~ . - pct.a.base)
tidy(rut.4) %>% arrange(p.value)
```

```
# A tibble: 6 x 5
  term             estimate std.error statistic     p.value
  <chr>               <dbl>     <dbl>     <dbl>       <dbl>
1 log(viscosity)  -0.552      0.0818     -6.75 0.000000448
2 pct.a.surf       0.593      0.225       2.63 0.0143
3 voids            0.200      0.121       1.66 0.109
4 (Intercept)     -2.08       1.61       -1.29 0.208
5 run              0.360      0.325       1.11 0.279
6 fines            0.0889     0.0870      1.02 0.316
```

- Again, `fines` is the one to go. (Output identical as it should be.)

# Take out fines:

```
rut.5 <- update(rut.4, . ~ . - fines)
tidy(rut.5) %>% arrange(p.value) %>% dplyr::select(term, p.
```

```
# A tibble: 5 x 2
  term                 p.value
  <chr>                  <dbl>
1 log(viscosity) 0.0000000559
2 pct.a.surf     0.0200
3 voids          0.0577
4 run            0.365
5 (Intercept)    0.375
```

Can't take out intercept, so `run`, with P-value 0.36, goes next.

# Take out run:

```
rut.6 <- update(rut.5, . ~ . - run)
tidy(rut.6) %>% arrange(p.value) %>% dplyr::select(term, p.
```

```
# A tibble: 4 x 2
  term               p.value
  <chr>                <dbl>
1 log(viscosity) 5.29e-19
2 pct.a.surf     1.80e- 2
3 voids          4.36e- 2
4 (Intercept)    4.61e- 1
```

Again, can't take out intercept, so largest P-value is for `voids`, 0.044. But this is significant, so we shouldn't remove `voids`.

## Comments

- Here we stop: `pct.a.surf`, `voids` and `log.viscosity` would all make fit significantly worse if removed. So they stay.
- Different final result from taking things out one at a time (top), than by taking out 4 at once (bottom):

```r
summary(rut.6)
```

```

Call:
lm(formula = log(rut.depth) ~ pct.a.surf + voids + log(visc
    data = asphalt)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.53548 -0.20181 -0.01702  0.16748  0.54707 

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    -1.02079    1.36430  -0.748   0.4608
```

# Comments on variable selection

- Best way to decide which $x$'s belong: expert knowledge: which of them should be important.
- Best automatic method: what we did, "backward selection".
- Do not learn about "stepwise regression"! **eg. here**
- R has function `step` that does backward selection, like this:

```
step(rut.2, direction = "backward", test = "F")
```

Gets same answer as we did (by removing least significant x).

- Removing non-significant $x$'s may remove interesting ones whose P-values happened not to reach 0.05. Consider using less stringent cutoff like 0.20 or even bigger.
- Can also fit all possible regressions, as over (may need to do `install.packages("leaps")` first).

# All possible regressions (output over)

Uses package `leaps`:

```
leaps <- regsubsets(log(rut.depth) ~ pct.a.surf +
                      pct.a.base + fines + voids +
                      log(viscosity) + run,
                    data = asphalt, nbest = 2)
s <- summary(leaps)
with(s, data.frame(rsq, outmat)) -> d
```

# The output

```r
d %>% rownames_to_column("model") %>% arrange(desc(rsq))
```

```
      model        rsq pct.a.surf pct.a.base fines voids log.viscosity. run
1  6  ( 1 ) 0.9609642          *          *     *     *              *   *
2  5  ( 1 ) 0.9608365          *                *     *              *   *
3  5  ( 2 ) 0.9593265          *          *     *     *              *
4  4  ( 1 ) 0.9591996          *                      *              *   *
5  4  ( 2 ) 0.9589206          *                *     *              *
6  3  ( 1 ) 0.9578631          *                      *              *
7  3  ( 2 ) 0.9534561          *                *                    *
8  2  ( 1 ) 0.9508647          *                                     *
9  2  ( 2 ) 0.9479541                                 *              *
10 1  ( 1 ) 0.9452562                                                *
11 1  ( 2 ) 0.8624107                                                    *
```

# Comments

- Problem: even adding a worthless x increases R-squared. So try for line where R-squared stops increasing "too much", eg. top line (just log.viscosity), first 3-variable line (backwards-elimination model). Hard to judge.
- One solution (STAC67): adjusted R-squared, where adding worthless variable makes it go down.
- `data.frame` rather than `tibble` because there are several columns in `outmat`.

# All possible regressions, adjusted R-squared

```r
with(s, data.frame(adjr2, outmat)) %>%
  rownames_to_column("model") %>%
  arrange(desc(adjr2))
```

```
      model     adjr2 pct.a.surf pct.a.base fines voids log.viscosity. run
1  3  ( 1 ) 0.9531812          *                      *              *    
2  5  ( 1 ) 0.9530038          *                *     *              *   *
3  4  ( 1 ) 0.9529226          *                      *              *   *
4  4  ( 2 ) 0.9526007          *                *     *              *    
5  6  ( 1 ) 0.9512052          *          *     *     *              *   *
6  5  ( 2 ) 0.9511918          *          *     *     *              *    
7  3  ( 2 ) 0.9482845          *                *                    *    
8  2  ( 1 ) 0.9473550          *                                     *    
9  2  ( 2 ) 0.9442365                                 *              *    
10 1  ( 1 ) 0.9433685                                                *    
11 1  ( 2 ) 0.8576662                                                    *
```

# Revisiting the best model

- Best model was our rut.6:

```
tidy(rut.6)
```

```
# A tibble: 4 x 5
  term            estimate std.error statistic  p.value
  <chr>              <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)       -1.02     1.36      -0.748 4.61e- 1
2 pct.a.surf         0.555    0.220      2.52  1.80e- 2
3 voids              0.245    0.116      2.12  4.36e- 2
4 log(viscosity)    -0.646    0.0288   -22.5   5.29e-19
```

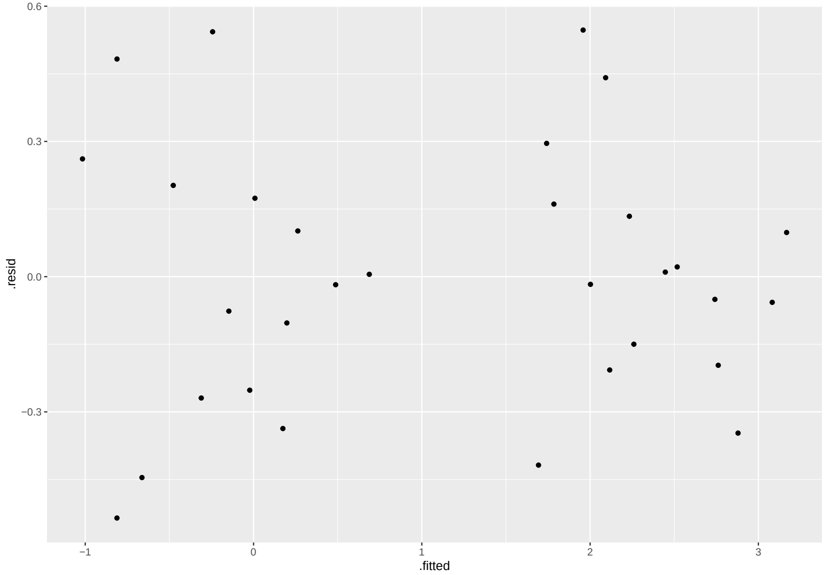
# Revisiting (2)

- Regression slopes say that rut depth increases as log-viscosity decreases, `pct.a.surf` increases and `voids` increases. This more or less checks out with out scatterplots against `log.viscosity`.
- We should check residual plots again, though previous scatterplots say it's unlikely that there will be a problem:

```
g <- ggplot(rut.6, aes(y = .resid, x = .fitted)) +
geom_point()
```

# Residuals against fitted values

```
g
```




```
ggplot(mut.6, aes(sample = .resid)) + stat_qq() + stat_qq_l
```

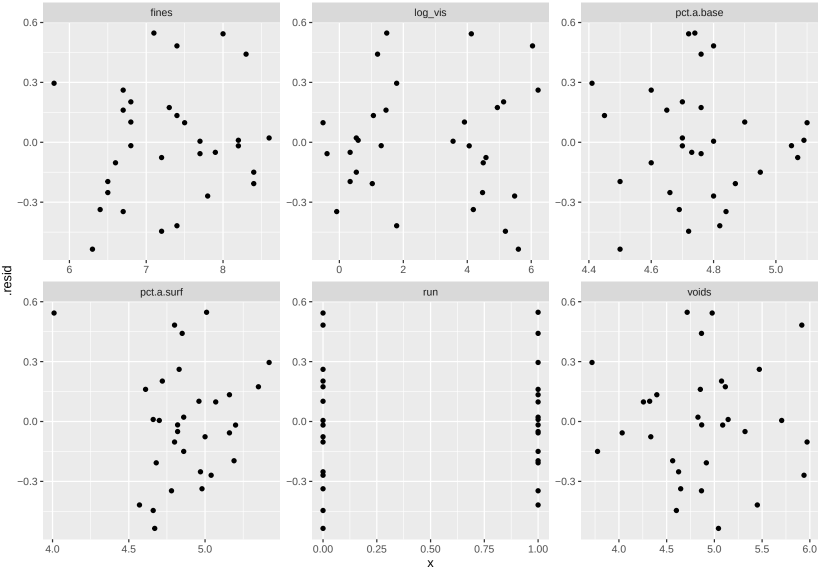
# Plotting residuals against x's

- Do our trick again to put them all on one plot:

```
augment(rut.6, asphalt) %>%
  mutate(log_vis=log(viscosity)) %>%
  pivot_longer(
    c(pct.a.surf:voids, run, log_vis),
    names_to="xname", values_to="x",
  ) %>%
  ggplot(aes(y = .resid, x = x)) + geom_point() +
  facet_wrap(~xname, scales = "free") -> g2
```

# Residuals against the x's

```
g2
```



# Comments

- None of the plots show any sort of pattern. The points all look random on each plot.
- On the plot of fitted values (and on the one of log.viscosity), the points seem to form a "left half" and a "right half" with a gap in the middle. This is not a concern.
- One of the pct.a.surf values is low outlier (4), shows up top left of that plot.
- Only two possible values of run; the points in each group look randomly scattered around 0, with equal spreads.
- Residuals seem to go above zero further than below, suggesting a mild non-normality, but not enough to be a problem.

# Variable-selection strategies

- Expert knowledge.
- Backward elimination.
- All possible regressions.
- Taking a variety of models to experts and asking their opinion.
- Use a looser cutoff to eliminate variables in backward elimination (eg. only if P-value greater than 0.20).
- If goal is prediction, eliminating worthless variables less important.
- If goal is understanding, want to eliminate worthless variables where possible.
- Results of variable selection not always reproducible, so caution advised.